Goverment Engineering College Jhalawar

Model Question Paper -1

Subject ÷ Engineering Mathematics - II

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Branch - Civil Engineering

Max. Marks-10

Semester - 2^{nd}

Do any four questions. Each question carry equal Marks.

Q-1 Solve the differential equation $(1+y^2)\,dx = (\tan^{-1}y - x)\,dy$.

Q-2 Solve the d.e. $2\sin x \dfrac{dy}{dx} - y\cos x = xy^3 e^x$

Q-3 Solve the d.e. $(x^2y - 2xy^2)\,dx - (x^3 - 3x^2y)\,dy = 0$

Q-4 Solve the d.e $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - 2y = x + \sin x$

Q-5 Solve the d.e $\dfrac{d^2y}{dx^2} - y = xe^x + e^x \cos^2 x$

Answer key

Model Question Paper 1 [Answer key]

Ans → 1 The given differential equation is

$$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$$

which is a linear differential equation.

where $P = \frac{1}{1+y^2}$, $Q = \frac{\tan^{-1}y}{1+y^2}$

i.e $I.F. = e^{\int P\,dy} = e^{\int \frac{1}{1+y^2} dy} = e^{\tan^{-1}y}$

Solution of given equation is

$$x \times I.F = \int Q \times I.F\, dy + c$$

$$x \times e^{\tan^{-1}y} = \int \frac{\tan^{-1}y}{1+y^2} \times e^{\tan^{-1}y} dy + c$$

Let $\tan^{-1}y = t \Rightarrow \frac{1}{1+y^2} dy = dt$

$$\therefore \quad x \times e^{\tan^{-1}y} = \int t e^t\, dt + c$$

$$x \times e^{\tan^{-1}y} = (te^t - e^t) + c$$

$$x \times e^{\tan^{-1}y} = (\tan^{-1}y\, e^{\tan^{-1}y} - e^{\tan^{-1}y}) + c$$

Ans-2 The given d.e can be written as

$$\frac{dy}{dx} - \frac{y}{2}\cot x = xy^3 e^x$$

$$\frac{1}{y^3}\frac{dy}{dx} - \frac{1}{2y^2}\cot x = \frac{xe^x}{2\sin x}$$

which is a bernouli's equation.

Let $\frac{1}{y^2} = v$, $-\frac{2}{y^3}\frac{dy}{dx} = \frac{dv}{dx}$

$$-\frac{1}{2}\frac{dv}{dx} - \frac{\cot x}{2} \times v = \frac{xe^x}{2\sin x}$$

$$\frac{dv}{dx} + \cot x \times v = -\frac{xe^x}{\sin x}$$

which is linear equation in v. $P = \cot x$, $Q = -\frac{xe^x}{\sin x}$

Sol. is → $V \times I.F = \int I.F \times Q \, dx + c$

where $I.F = e^{\int P dx} = e^{\int \cot x \, dx} = e^{\log(\sin x)} = \sin x$

$$v \times \sin x = -\int \cancel{\sin x} \times \frac{xe^x}{\cancel{\sin x}} dx + c$$

$$\sin x \times v = -\int xe^x dx + c$$

$$\sin x \times v = -xe^x + e^x + c$$

$$\sin x \times \frac{1}{y^2} = e^x(1-x) + c$$

Ans → 3 Comparing the given equation with $Mdx + Ndy = 0$

we have $M = (x^2y - 2xy^2)$, $N = -(x^3 - 3x^2y)$

$$\frac{\partial M}{\partial y} = x^2 - 4xy, \quad \frac{\partial N}{\partial x} = -[3x^2 - 6xy]$$

$\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ so the given d.e is not a exact d.e.

Since the given equation is homogeneous d.e.

we take a $I.F = \frac{1}{Mx + Ny} = \frac{1}{x^3y - 2x^2y^2 - x^3y + 3x^2y^2}$

$$= \frac{1}{x^2y^2} \neq 0$$

On multiplying the given equation by $\frac{1}{x^2y^2}$

$$\left(\frac{1}{y}-\frac{2}{x}\right)dx - \left(\frac{x}{y^2}-\frac{3}{y}\right)dy = 0$$

which is a exact differential equation.

Sol. is Step. 1: $u = \int M\,dx = \int\left[\frac{1}{y}-\frac{2}{x}\right]dx = \frac{x}{y} - 2\log x$

Step. 2: $\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left[\frac{x}{y} - 2\log x\right] = -\frac{x}{y^2}$

Step. 3: $N - \frac{\partial u}{\partial y} = -\frac{x}{y^2} + \frac{3}{y} + \frac{x}{y^2} = \frac{3}{y}$

Step 4: $V = \int\left[N - \frac{\partial u}{\partial y}\right]dy = \int \frac{3}{y}\,dy = 3\log y$

Step 5: Sol. is $U + V = C$

$$\frac{x}{y} - 2\log x + 3\log y = C$$

Ans-4. The auxiliary equation is $m^2 + m - 2 = 0$

$\Rightarrow (m+2)(m-1) = 0$

$\Rightarrow m = -2, 1$

$$C.F = c_1 e^{-2x} + c_2 e^{x}$$

$$P.I = \frac{1}{(D^2+D-2)}\, x + \sin x = \frac{1}{D^2+D-2} \times x + \frac{1}{D^2+D-2} \times \sin x$$

$$= -\frac{1}{2}\,\frac{1}{\left[1 - \frac{D^2+D}{2}\right]}\, x + \frac{(D+3)}{(D-3)(D+3)} \sin x$$

$$= -\frac{1}{2}\left[1 + \frac{D}{2}\right] x + \frac{(D+3)}{(D^2-9)} \sin x$$

$$= -\frac{1}{2}\left[x + \frac{1}{2}\right] + \frac{\cos x + 3\sin x}{-10}$$

$$y = C.F + P.I = c_1 e^{-2x} + c_2 e^{x} - \frac{x}{2} - \frac{1}{4} - \frac{1}{10}(\cos x + 3\sin x)$$

Ans:- 5 The given d.e is $(D^2-1)y = xe^x + \cos^2 x$

Therefore A.E is $m^2-1=0 \Rightarrow m=\pm 1$

$$C.F = c_1 e^{-x} + c_2 e^{x}$$

$$P.I = \frac{1}{D^2-1}(xe^x + \cos^2 x)$$

$$P.I = \frac{1}{D^2-1} xe^x + \frac{1}{D^2-1}\cos^2 x$$

$$P.I = e^x \frac{1}{(D+1)^2-1} x + \frac{1}{D^2-1} \times \frac{(1+\cos 2x)}{2}$$

$$P.I = e^x \frac{1}{D(D+2)} x + \frac{1}{2}\left[\frac{1}{D^2-1} \times 1 + \frac{1}{D^2-1} \times \cos 2x\right]$$

$$P.I = e^x \frac{1}{2D}\left[1+\frac{D}{2}\right]^{-1} x + \frac{1}{2}\left[-(1-D^2)^{-1} \times 1 + \frac{1}{-4-1} \times \cos 2x\right]$$

$$P.I = e^x \frac{1}{2D}\left(x - \frac{1}{2}\right) - \frac{1}{2} - \frac{\cos 2x}{10}$$

$$P.I = \left(\frac{x^2}{4} - \frac{x}{4}\right) e^x - \frac{1}{2} - \frac{\cos 2x}{10}$$

Sol. $y = C.F + P.I$

$$y = c_1 e^{-x} + c_2 e^x + \left(\frac{x^2}{4} - \frac{x}{4}\right) e^x - \frac{1}{2} - \frac{\cos 2x}{10}$$